Çankaya University
Department of Mathematics

Math 452, Topology
Final Exam

May 28, 2018

Name: ____________________________________

Number: ____________________________________

Question 1 (20 points)

Show that the following set is **compact, connected, closed and bounded**.

$$A = \{(t^2, t^3, e^t, e^{2t}) \in \mathbb{R}^4 : -4 \leq t \leq 4\}$$

Question 2 (20 points)
Let $X = \{1, 2, 3, 4, 5\}$ and let τ_1 and τ_2 be two topologies on X given by

$$\tau_1 = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}, \{1, 3, 4\}, \{1, 2, 3, 4\}, \{1, 2, 3, 5\}, X\}$$

$$\tau_2 = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}, \{1, 3, 4\}, \{1, 2, 3, 4\}, \{1, 3, 4, 5\}, X\}$$

Let $X_1 = (X, \tau_1)$ and $X_2 = (X, \tau_2)$ be the corresponding topological spaces.

a) Is there a proper subset of X which is both open and closed in X_1?

b) Are X_1 and X_2 homeomorphic? Describe your answer.

Question 3 (20 points) What is wrong with the following argument? Describe your answer.

The family of open intervals $G = \{(-k, k) : k = 1, 2, 3, \ldots\}$ is an open covering of the interval $(0, 4)$. There exists a finite subset of G (for example, the interval $(-6, 6)$) which covers $(0, 4)$. Hence, $(0, 4)$ is compact.

Question 4 (20 points)
Let $X = [0, 2) \subseteq \mathbb{R}$ and let $Y = (-3, 5] \subseteq \mathbb{R}$. Find a homeomorphism from the set X to the set Y. Justify your answer.

Question 5 (20 points)
Consider $X = [0, 2] \setminus \{1\}$ as a subspace of the real line $\mathbb{R}$.
Show that the subset $[0, 1) \subset X$ is both open and closed in X.