Çankaya University
Department of Mathematics

Math 452, Topology
Midterm Exam

April 27, 2018

Name: ________________________________

Number: ________________________________

Question 1 (12 points)

a) Give the definition of the subspace topology.

b) State Urysohn's lemma.

c) Give the definition of the product topology.

d) Give the definition of the Normal space.

Question 2 (15 points)
State whether the following propositions are **true** or **false**. If they are true DO NOT PROVE them. If they are false give a counterexample.

(a) A continuous bijection from a compact topological space onto a Hausdorff topological space is necessarily a homeomorphism.

(b) *Every subspace of a compact space is also compact.*

(c) Let X and Y be topological spaces and $f : X \to Y$ be a continuous function. Let A be a subset of X. Then, if A is compact, $f(A)$ is also compact.

(d) *Every convergent sequence in a Hausdorff space has a unique limit.*

(e) Let X and Y be topological spaces and $f : X \to Y$ be a continuous function. Let A be a subset of X. Then, if A is open, $f(A)$ is also open.

Question 3 (20 points)
Prove that, a function $f : X \to Y$ is continuous if and only if for every subset $A \subseteq X$, $f(\overline{A}) \subseteq \overline{f(A)}$.

Question 4 (13 points)
Let $X = \{1, 2, 3, 4, 5\}$ and let $\mathscr{A} = \{\{1, 3, 5\}, \{2, 3\}, \{4\}\}$. Find the topology on X generated by $\mathscr{A}$.

Question 5 (20 points)

Show that $G = \{(x, y) \in \mathbb{R}^2 : x^2y - 5xy^3 \notin \mathbb{Z}\}$ is an open subset of $\mathbb{R}^2$ with usual topology.

Question 6 (20 points)

Choose and solve ONLY ONE of the following:

a) Show that every closed subspace of a compact space is compact

b) Show that a compact subset of a Hausdorff space is closed.