

Çankaya University
Department of Mathematics

Math 452, Topology
Resit Exam

June 13, 2018

Name: _____

Number: _____

Question 1 (20 points)

- a) Give the definition of a Hausdorff space.
- b) If $f : X \rightarrow Y$ is one-to-one and continuous and Y is Hausdorff, is it necessarily true that X is Hausdorff? If so, provide a proof, if not, provide a counterexample.

Question 2 (20 points)

Let (X, τ) be a topological space with $A, B \subseteq X$. Prove that

$$\overline{A \cup B} = \overline{A} \cup \overline{B}$$

Question 3 (20 points) a) Let the set $X = \{1, 2, 3\}$ and the topology $\tau = \{\emptyset, \{1\}, \{1, 2\}, X\}$ on X is given. Describe the closures of the sets $\{2\}$ and $\{3\}$.

- b) True or False: If A is a subset of the set of real numbers \mathbb{R} with the usual topology such that \overline{A} is connected, then A is connected. Justify your answer.

Question 4 (20 points)

Show that the following set is **compact, connected, closed and bounded**.

$$A = \left\{ \left(\frac{1}{1+t^2}, 5, e^{-2t} \right) \in \mathbb{R}^3 : -5 \leq t \leq 5 \right\}$$

Question 5 (20 points)

Show that $A = \{(x, y) \in \mathbb{R}^2 : x^2 + \sin y \leq 1\}$ is not compact.